

THE GENERALIZED COMPOSITE COMMODITY THEOREM AND FOOD DEMAND ESTIMATION

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This article reports tests of aggregation over consumer food products and estimates of aggregate food demand elasticities. Evidence that food demand variables follow unit root processes leads us to build on and simplify existing tests of the Generalized Composite Commodity Theorem. We compute food demand elasticities using a method of cointegration that is shown to apply to a convenient but nonlinear functional form. Estimates are based on consumer reported expenditure data rather than commercial disappearance data.

Key words: cointegration, food demand, generalized composite commodity theorem, nonlinear models.

Appropriately specified models of consumer demand are central to market analysis. It has been established that a valid equilibrium for markets characterized by diverse firms or diverse consumer products depends on downward-sloping composite demand curves (Heiner, Braulke, Wohlgenant) or on valid indirect utility functions (Chavas and Cox). Given the large number of consumer food products, food demand and price analysis must be implemented at some level of product aggregation. Improper aggregation can lead to biased estimates of welfare loss associated with public policies, biased estimates of consumer and derived demand elasticities, and misleading tests of market power. Testing for consistent aggregation over food products and estimating aggregate consumer food demand elasticities are the subjects of this article.

By far, the most common justification for aggregation has been separable preferences. One reason for its popularity has been the lack of a viable alternative. Tests of the Composite Commodity Theorem (Hicks, Leontief, Lewbel) are always rejected because it restricts relative prices within a group to remain fixed over time. On the other hand, aggregation based on weak separability is often rejected (Diewert and Wales, Eales and Unnevehr).

The Generalized Composite Commodity Theorem (GCCT) justifies aggregation under milder conditions (Lewbel). It relaxes the restriction of constant relative prices within groups by strengthening the requirement that independence holds across all groups. Furthermore, the GCCT simplifies tests for weakly separable preferences.

This article tests for valid aggregation of consumer food products and reports estimates of food demand elasticities. As in previous studies, we find evidence that food demand variables contain unit roots, so tests for valid aggregates involve tests for spurious regressions. By building on the methodology found in previous studies we simplify tests for valid aggregation. In addition, we present estimates of food demand elasticities and tests for weak separability based on recent developments in the theory of nonlinear nonstationary regressions.

Theory

The GCCT is a stochastic theory of aggregation over diverse consumer products. It maintains that n elementary share equations are functions of logged elementary prices, \mathbf{r} , and logged income, z . Following Lewbel, let w_i ($i = 1, \dots, n$) denote the i th elementary budget share and let E denote the mathematical expectations operator. Then $g_i(\mathbf{r}, z) \rightarrow w_i$ ($i = 1, \dots, n$) such that

$$(1) \quad w_i = g_i(\mathbf{r}, z) + e_i \quad \text{where} \\ E(e_i | \mathbf{r}, z) = 0 \Rightarrow E(w_i | \mathbf{r}, z) = g_i(\mathbf{r}, z).$$

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Since the g_i form a valid elementary demand system, they satisfy adding-up ($\sum g_i = 1$), homogeneity ($g_i(\mathbf{r} - k, z - k) = g_i(\mathbf{r}, z)$ for all i), and Slutsky symmetry (i.e., $(\partial g_k / \partial r_j) + (\partial g_k / \partial z) g_j = (\partial g_j / \partial r_k) + (\partial g_j / \partial z) g_k$). The compensated demands satisfy negative semidefiniteness.

The theory also maintains the existence of a system of stochastic composite share equations. The $M(< n)$ composite shares $W_I \equiv \sum_{i \in I} w_i$ ($I = 1, \dots, M$) are functions of logged income z and logged composite prices \mathbf{R} , or $G_I(\mathbf{R}, z) \rightarrow W_I$ ($I = 1, 2, \dots, M$). In particular,

$$(2) \quad W_I = G_I(\mathbf{R}, z) + u_I \quad \text{where}$$

$$E(u_I | \mathbf{R}, z) = 0 \Rightarrow G_I(\mathbf{R}, z) = E(W_I | \mathbf{R}, z).$$

The orthogonality of the model errors of (1) and (2) ensure that $g_i(\mathbf{r}, z)$ and $G_I(\mathbf{R}, z)$ are optimal predictors of elementary and composite shares, respectively.

These model errors are related. Following Lewbel, let $G_I^*(\mathbf{r}, z)$ denote the sum of the conditional means of the elementary demands for group I , so that $G_I^*(\mathbf{r}, z) \equiv \sum_{i \in I} g_i(r, z)$. Also define $\rho_i \equiv r_i - R_I$ as the i th relative price so the vector of all relative prices is $\boldsymbol{\rho} = \mathbf{r} - \mathbf{R}^*$ where \mathbf{R}^* denotes the n -vector of group prices with R_I in row i and in every row $i \in I$. This implies

$$(3) \quad u_I = \sum_{i \in I} e_i + G_I^*(\boldsymbol{\rho} + \mathbf{R}^*, z) - G_I(\mathbf{R}, z)$$

which shows the composite model errors are correlated with relative elementary prices.

Lewbel shows that valid aggregation is obtained when the vector of all relative prices is distributed independently of the vector of composite prices and income. This implies

$$(4) \quad G_I(\mathbf{R}, z) = \int G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) dF(\boldsymbol{\rho})$$

which states that the conditional expectation of the I th composite share equals an unconditional expectation of sums of the elementary demand functions in the I th composite. Lewbel uses (4) to obtain three results that relate directly to demand system estimation. First, $G_I(\mathbf{R}, z)$ ($I = 1, 2, \dots, M$) is a valid system of composite demand equations because this system inherits the adding up, homogeneity, and nearly (or in some cases exactly) inherits Slutsky symmetry from the elementary demands. Second, the demand elasticities of $G_I(\mathbf{R}, z)$ are best, unbiased estimates of within-group sums of elementary demand elasticities. Third, (3)

and (4) imply that $(G_I^* - G_I)$ is a bias term that arises from aggregation and this term is a function of $\boldsymbol{\rho}$. Because u_I contains this bias, the errors of a composite demand system justified by the GCCT will be correlated with relative prices. If instead the demand system is based on weakly separable preferences, $G_I^* = G_I$ so $u_I = \sum_{i \in I} e_i$ and composite demand errors will not be correlated with relative prices.

In time series theory, the restriction that $\boldsymbol{\rho}_t$ is distributed independently of $\mathbf{q}_t \equiv [\mathbf{R}'_t, z_t]'$ imposes restrictions on the correlation of an infinite number of random variables. For example, if $\boldsymbol{\rho}_t$ and \mathbf{q}_t are in-deterministic stationary processes they would satisfy

$$\boldsymbol{\rho}_t = \sum_{s=0}^{\infty} \mathbf{C}_s \mathbf{V}_{t-s} \quad \text{and} \quad \mathbf{q}_t = \sum_{s=0}^{\infty} \mathbf{D}_s \mathbf{V}_{t-s}$$

where $\sum_{s=0}^{\infty} |\mathbf{C}_s| < \infty$, $\sum_{s=0}^{\infty} |\mathbf{D}_s| < \infty$, and $(\mathbf{V}_t \mathbf{U}_t)$ are *iid* normal with mean zero. In this case independence requires $E(\mathbf{V}_t \mathbf{U}_s) = 0$ for all t, s . An implication of vector independence is $E(\boldsymbol{\rho}_t | \mathbf{q}_t) = E(\boldsymbol{\rho}_t)$ or that \mathbf{q}_t provides no information about $\boldsymbol{\rho}_t$. These restrictions may be difficult or impossible to test.

This may be why tests of the GCCT have focused on tests for linear relationships. If P denotes the linear projection operator, valid aggregation means $P(\boldsymbol{\rho}_t | \mathbf{q}_t)$ is not a linear function of \mathbf{q}_t and failure to reject such tests is taken as support for valid aggregation (Lewbel; Davis; Davis, Lin, and Shumway; Asche, Bremmes, and Wessells). Because evidence has suggested $\boldsymbol{\rho}_t$ and \mathbf{q}_t are often unit root processes, aggregation tests have been based on tests of spurious regressions.

Phillips shows that a simple linear regression constructed from two independently distributed and integrated time series behaves like a model constructed from two non-cointegrated series. That is, Phillips shows correlation between the stationary components of the two series will not affect the asymptotic behavior of the regression. It is significant for our work that Phillips extends this result to multiple regression models, so that if $\rho_{it} = P(\rho_{it} | \mathbf{q}_t) + v_{it}$ is a spurious regression, the model behaves as though ρ_{it} is distributed independently of \mathbf{q}_t . This suggests tests of the GCCT that have been based on a large number of simple regression models can be simplified by basing them instead on multiple regression models.

In particular, if \mathbf{q}_t is an integrated vector we can, in a straightforward manner, compute a test of the aggregation scheme. An approach is

to compute Engle-Granger tests of no cointegration for each of the individual ρ_i multiple regressions for which ρ_i is an integrated variable, and then follow Davis, Lin, and Shumway. That is, based on the individual tests use the Holm procedure to test the family-wise hypothesis that each integrated element of ρ is jointly spuriously related to q . This approach differs from the "grand test" proposed by Davis because it represents a test of the null of valid aggregation. Moreover, our reading of Huang leads us to conclude that the power problems associated Engle-Granger tests are no more severe for regressions with a large number of regressors than they are for a small number of regressors.

A finding of valid aggregation means a composite demand system is associated with this aggregation scheme. Moreover, if demand variables follow unit root processes, we expect the demand equations to be cointegrated (Karagiannis and Mergos). Because of the interest in demand elasticities, and because a number of useful functional forms used in empirical demand work are nonlinear, it is desirable to apply cointegration methods to nonlinear demand systems.

Cointegration and Nonlinear Share Equation Systems

We maintain that composite shares of a valid demand system are adequately described by the semi-flexible almost ideal (SAI) demand system (Moschini). The SAI demand system is a re-parameterization of the Almost Ideal (AI) demand system (Deaton and Muellbauer). Thus, it describes nonlinear Engle curves, defines community income and exact nonlinear aggregation over consumers, and defines budget shares and income elasticities for income inelastic goods, such as food, that decline as incomes rise (Moschini). Moreover, the SAI demand system saves degrees of freedom while maintaining curvature at a point in the data. In this section we show that a version of the nonlinear SAI demand system can be estimated using cointegration methods.

Recall that W_I denotes the I th composite consumer budget share, z the log of income, and R_J the log of the J th composite price. If we let e_i denote the I th model error, the AI model is

$$(5) \quad W_I = \alpha_I + \sum_{J=1}^M \gamma_{IJ} R_J + \beta_I (z - \log P) + e_I \quad (I = 1, \dots, M)$$

$$(6) \quad \log P = \alpha_0 + \sum_{I=1}^M \alpha_I R_I + 1/2 \sum_{I=1}^M \sum_{J=1}^M R_I R_J$$

with Slutsky-substitution terms

$$(7) \quad S_{IJ} = [x/(p_I p_J)] [\gamma_{IJ} + W_I W_J - \delta_{IJ} w_I + \beta_I \beta_J (z - \log P)]$$

where $\delta_{II} = 1$ for $I = J$ and $\delta_{IJ} = 0$ for $I \neq J$. Moschini notes that if $\alpha_0 = 0$ and price and income variables are deflated by their sample means, then at the sample mean $p_I = x = 1$, α_I is the I th budget share, $\varepsilon_{IJ} = (1/\alpha_I)(\gamma_{IJ} - \beta_I \alpha_I) - \delta_{IJ}$ is a cross-price elasticity of demand, $\eta_I = (\beta_I/\alpha_I) + 1$ is the income elasticity of demand, and the Slutsky substitution terms are

$$(8) \quad \theta_{IJ} = \gamma_{IJ} + \alpha_I \alpha_J - \delta_{IJ} \alpha_I.$$

The SAI model can be used to estimate demand elasticities conditioned on curvature imposed at the mean (or any other point) of the data. By setting $\Theta = [\Theta_{IJ}] = -T'T$ where $T = [\tau_{IJ}]$ is upper triangular and Θ is less than full rank, Moschini restricts the rank of Θ by setting the last number of rows equal to zero. For example if Θ is a 5×5 matrix, setting the last two rows of T to zero restricts Θ to a matrix of rank three. Such restrictions allow the parameters of

$$(9) \quad W_I = \alpha_I + \alpha_I R_I - \alpha_I \sum_{J=1}^M \alpha_J R_J - \sum_{S=1}^I \tau_{SI} R_J \sum_{J=S}^{M-1} \tau_{SJ} (R_J - R_M) + \beta_I z - \beta_I \log P + e_I \quad (I = 1, \dots, M-1)$$

$$(10) \quad \log P = \sum_{J=1}^M \alpha_J R_J - 1/2 \left[\sum_{J=1}^M \alpha_J R_J \right]^2 + 1/2 \sum_{J=1}^M \alpha_J (R_J)^2 - 1/2 \sum_{S=1}^{M-1} \left[\sum_{J=S}^{M-1} \tau_{SJ} (R_J - R_M) \right]^2$$

identify the parameters of (5) and (6). Equations (9) and (10) represent the SAI demand model.

The task is to estimate this nonlinear system assuming each share equation is an integrated regression with stationary model errors. The estimator developed by Chang, Park, and Phillips applies to a class of nonlinear, single equation cointegrated models. To describe this class in detail, let (9) be represented as

$$(11) \quad W_t = \alpha + q(x_t, \beta) + e_t$$

where $x'_t = [x_{1t}, x_{2t}, \dots, x_{kt}]$ is a k -vector of integrated stochastic regressors. Chang, Park, and Phillips show their estimator applies to nonlinear models such as (11) when model errors are serially uncorrelated (i.e., e_t is a martingale difference series) and $q(x_t, \beta)$ satisfies additive separability. They show that if $q(x_t, \beta)$ consists of k additive terms for $\beta = [\beta'_1, \beta'_2, \dots, \beta'_k]$, additive separability requires

$$q(x_t, \beta) = \sum_{i=1}^k q_i(x_{it}, \beta_i).$$

Thus, additive separability permits only one integrated regressor per additive term.

For an SAI demand equation, it is seen that if $\log P$ could be treated as one integrated regressor, (9) would satisfy additive separability. The problem lies with $\log P$. With the exception of the first term, every term in (10) involves two integrated variables, so (9) and (10) violate additive separability. However, the approximation (Deaton and Muellbauer)

$$(10') \quad \log P \approx \sum_{J=1}^M W_J R_J$$

does treat $\log P$ as a single variable so (9) and (10') satisfy additive separability. Moschini notes that (9) and (10') represent a valid form of the SAI model, and demand system estimation in this study is based on (9) and (10').

For integrated regressions, cointegration does not ensure econometric exogeneity. The consequence for estimation of linear, single equation cointegrated regressions is that ordinary least squares (OLS) estimates are biased and even though cointegration means this bias disappears in large samples this bias injects nuisance parameters into the distributions of OLS estimates. The result is that standard t - and F -tests are misleading even in large samples. When econometric exogeneity is achieved, the bias and the nuisance parameters disappear so that OLS estimates are normally distributed, and t - and

F -tests provide correct inference. The fully modified (FM) estimator transforms single equation linear models in such a way that the transformed model satisfies econometric exogeneity (Phillips and Hansen).

Chang, Park, and Phillips show similar issues generally arise in nonlinear cointegrated regressions, and so derive an FM estimator for single nonlinear cointegrated regressions. To describe their estimator, it is assumed that model errors of (11) form a martingale difference series and the regression function, q , satisfies additive separability. Let $\Delta x_t = v_t$ and $E(v_t) = 0$ so $[e_t, v_t]$ form a linearly indeterministic stationary vector. The covariance generating function (evaluated at frequency 1) of this vector is

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} = \sum_{k=-\infty}^{\infty} \begin{bmatrix} E(e_t e'_{t-k}) & E(e_t v'_{t-k}) \\ E(v_t e'_{t-k}) & E(v_t v'_{t-k}) \end{bmatrix}.$$

Since v_t is a stationary, serially correlated series, it satisfies $v_t = \sum_{i=0}^{\infty} \pi_i \varepsilon_{t-i}$ where ε_t is a serially uncorrelated *iid* vector process with $\sum_{i=0}^{\infty} |\pi_i| < \infty$. If the model errors are also stationary and serially correlated they satisfy $e_t = \sum_{i=0}^{\infty} \phi_i \zeta_{t-i}$ with $\sum_{i=0}^{\infty} |\phi_i| < \infty$ (where ζ_t may be contemporaneously but not serially correlated with ε_t). In this case

$$\begin{aligned} \Sigma_{12} &= \sum_{k=-\infty}^{\infty} E(e_t v'_{t-k}) \\ &= \sum_{k=-\infty}^{\infty} E \left(\sum_{i=0}^{\infty} \phi_i \zeta_{t-i} \right) \\ &\quad \times \left(\sum_{j=0}^{\infty} \pi_j \varepsilon_{t-j-k} \right) \neq 0 \end{aligned}$$

means exogeneity is violated. However, for serially uncorrelated errors $\phi_i = 0$ for $i > 0$, and

$$\begin{aligned} \Sigma_{12} &= \sum_{k=-\infty}^{\infty} E(e_{tj} v'_{t-k}) \\ &= \sum_{k=-\infty}^{\infty} E(\phi_0 \zeta_t) \left(\sum_{j=0}^{\infty} \pi_j \varepsilon_{t-j-k} \right) \\ &= E(\phi_0 \pi(1) \zeta_t \varepsilon_t) \\ &= E(e_{tj} v_t) \equiv \Sigma_{12}^0 \end{aligned}$$

where $\pi(1) = \sum_{j=0}^{\infty} \pi_j$. Hence violations of econometric exogeneity arise in models with serially uncorrelated errors only because of the

presence of contemporaneous correlation between e_t and v_t . Define $\Sigma_{22}^0 = E(v_t v_t')$ so that subtracting $\Sigma_{12}^0 (\Sigma_{22}^0)^{-1} v_t$ from both sides of (11) gives

$$(11') \quad W_t^* = \alpha + q(x_t, \beta) + e_t^*$$

where $W_t^* = W_t - \Sigma_{12}^0 (\Sigma_{22}^0)^{-1} v_t$, $e_t^* = e_t - \Sigma_{12}^0 (\Sigma_{22}^0)^{-1} v_t$ so that

$$\begin{aligned} & \sum_{k=-\infty}^{\infty} E(e_t^* v_{t-k}) \\ &= E(e_t v_t') - \Sigma_{12}^0 (\Sigma_{22}^0)^{-1} E(v_t, v_t') \\ &= \Sigma_{12}^0 \Sigma_{12}^0 = 0 \end{aligned}$$

and econometric exogeneity obtains. This means nonlinear least squares (NLS) estimates of (11') are consistent, efficient, and normally distributed. Furthermore, consistent estimates of Σ_{12}^0 and Σ_{22}^0 obtain by applying NLS to (11) in the first stage. Chang, Park, and Phillips refer to this estimator as the *efficient nonstationary nonlinear least squares* (EN-NLS) estimator.

To estimate demand elasticities of an SAI demand model using the EN-NLS estimator, it must be expanded to a systems estimator. It should be recognized that when estimating systems of cointegrated regressions, a violation of econometric exogeneity means SUR or nonlinear SUR (NSUR) yield inconsistent estimates.¹ The implication for FM system estimation is that single-equation estimators such as OLS, NLS, or EN-NLS rather than multivariate estimators such as SUR or NSUR must be used in estimating the model in the first stage. The SUR or NSUR estimator is then applied to the transformed model in the second stage.

More specifically, represent the $M - 1$ SAI composite demand equations as

$$(12) \quad W_{Jt} = \alpha_J + q_J(x_t, \beta_J) + e_{Jt} \quad (J = 1, 2, \dots, M - 1)$$

where W_{Jt} is the J th composite budget share in time t , x_t is a vector of integrated prices and income, q_J is additively separable, e_{Jt} is the J th element of the $M - 1$ vector e_t which is drawn from a martingale difference series.

Let $v_t = \Delta x_t$, denote the vector of first differences of non-redundant regressors in the system with $E(\Delta x_t) = 0$. Since v_t is a stationary vector series, it satisfies $v_t = \sum_{i=0}^{\infty} \pi_i \varepsilon_{t-i}$ and in general e_t satisfies $e_t = \sum_{i=0}^{\infty} P_i \zeta_{t-i}$. But because $E(e_t | e_{t-1}, e_{t-2}, \dots) = 0$, $\text{Var}(e) = P_o \Lambda P_o'$ where $\Lambda = E(\zeta_t \zeta_t')$ so $e_t = P_o \zeta_t$ where ζ_t is a serially uncorrelated *iid* vector process that can be contemporaneously but not serially correlated with ε_t . Then

$$\begin{aligned} \Sigma_{12} &= \Sigma_{k=-\infty}^{\infty} E(e_t v_{t-k}') \\ &= \sum_{k=-\infty}^{\infty} E(P_o \zeta_t) \left(\sum_{j=0}^{\infty} \pi_j \varepsilon_{t-j-k} \right) \\ &= E[P_o \zeta_t \pi(1) \varepsilon_t] = E(e_t v_t) \equiv \Sigma_{12}^0 \end{aligned}$$

and violation of strict econometric exogeneity in a system of cointegrated regressions derives only from the presence of contemporaneous correlation between e_t and v_t when model errors are serially uncorrelated. Define $\Sigma_{22}^0 = E(v_t v_t')$ and let $\omega_{120}^{(J)}$ represent the J th row of Σ_{12}^0 so that subtracting $\omega_{120}^{(J)} (\Sigma_{22}^0)^{-1} v_t$ from (12) gives

$$(12') \quad W_{Jt}^* = \alpha_J + q_J(x_t, \beta_J) + e_{Jt}^* \quad (J = 1, 2, \dots, M - 1)$$

where $W_{Jt}^* = W_{Jt} - \omega_{120}^{(J)} (\Sigma_{22}^0)^{-1} v_t$ and $e_{Jt}^* = e_{Jt} - \omega_{120}^{(J)} (\Sigma_{22}^0)^{-1} v_t$. This means

$$\begin{aligned} & \sum_{k=-\infty}^{\infty} E(e_{Jt}^* v_t') \\ &= E(e_{Jt} v_t') - \omega_{120}^{(J)} (\Sigma_{22}^0)^{-1} E(v_t v_t') \\ &= \omega_{120}^{(J)} - \omega_{120}^{(J)} = 0 \end{aligned}$$

and econometric exogeneity is achieved. This means that given consistent estimates of Σ_{12}^0 estimates of β from (12') are consistent and normally distributed. It should be noted this nonlinear estimator takes the same form as the FM estimator for linear systems (Moon).

There are two more points worth mentioning. First, it is well known that because $\sum_{J=1}^M W_{Jt} = 1$, the error covariance matrix is singular and the model is estimated using $M - 1$ equations (Berndt and Savin). When cross-equation restrictions are imposed in the first stage of a mean distance estimator (e.g., SUR) as they are for the SAI model, the estimates will not be invariant to the omitted equation unless one uses a first-stage-weight matrix that

¹ The reason is for a system of cointegrated regressions with correlation across the model errors and nonzero correlation with the regressors, information on unit root variables in the system is transmitted to the equations across the system and the SUR estimator does not weight that information properly. The result is a bias term associated with a SUR or NSUR estimator that may not disappear asymptotically (Park and Ogaki).

treats equations symmetrically (see Chavas and Sergerson). The problem is these matrices contain nonzero off diagonal elements and this leads to inconsistent estimates of cointegrated systems for the same reasons SUR or NSUR yields inconsistent estimates (see footnote). In this study we use the identity as the first-stage-weight matrix, and recognize the estimates are consistent but not invariant to the equation omitted.

Second, Chavas and Sergerson note that if the model errors are included in the specification of share equations, as they are in (12), they also enter the indirect utility function and so can lead to heteroskedastic errors. They recommend applying a GLS transformation that accounts for heteroskedasticity prior to estimation. However, cointegration theory is based on a data-generating process in which partial sums are distributed like continuous time, Brownian motion variables (Phillips and Durlauf). This automatically allows for heteroskedastic errors, so the only transformations that are necessary are those ensuring econometric exogeneity.

Empirical Results

This section reports a test of valid aggregation of nineteen elementary at-home food products, estimates of composite food demand elasticities, and tests for weakly separable preferences.

We propose the nineteen food products be aggregated into the following five at-home food composites. The cereal and bakery composite includes all cereal and bakery products. The meat composite includes beef, pork, other meat, poultry, and fish and seafood. The dairy composite includes fluid milk, butter, cheese, and ice cream. The fruit and vegetable composite includes fresh fruit, fresh vegetables, and processed fruit and vegetables. The other food-at-home composite includes sugar and sweets, fats and oils, nonalcoholic beverages, eggs, and miscellaneous foods. Food-away from home and nonfood are treated as valid composites.

Estimates of U.S. quarterly budget shares, after-tax income, and community income are computed from 1982.1 to 2000.4 using weighted sums of household expenditures reported in the diary section of the Consumer Expenditure Survey (CES) (U.S. Department of Labor), with weights supplied by the CES. Quarterly after-tax income is constructed as the U.S. annual estimate divided by four, and nonfood expenditure is the difference between

the quarterly after-tax income and the sum of away-from-home and at-home food expenditures. The quarterly budget share for the I th composite, w_I , is computed as the ratio of the expenditure for the i th good-to-after tax income. If x_h denotes the (weighted) total expenditures for household h and k_h denotes the number of members in household h , then community income (per capita) associated with PIGLOG preferences (Muellbauer) is $x_0 = \exp[\sum_h x_h \log(x_h/k_h) / \sum_h x_h]$. If m_x denotes the sample mean of x_0 , $z \equiv \log(x_0/m_x)$ is used in estimation.

Quarterly Laspeyres price indices (1982–84 = 1.0), P_J , $J = 1, \dots, 5$ are constructed for the five at-home categories using the nineteen elementary prices, p_j , with expenditure weights constructed from the expenditure data. Logs of mean-deflated prices are used in testing and estimation. Specifically, if M_J denotes the sample mean of P_J and m_j denotes the sample mean of p_j , then the J th log mean-deflated composite price is $\log(P_J/M_J) \equiv R_J$ and the j th relative elementary price is $\log(p_j/m_j) - R_J \equiv \rho_j$ for every $j \in J$.

Table 1 reports unit root tests on relative prices, composite prices, and community income. They indicate that at the $\alpha = 0.10$ level of significance most of the twenty-seven elements of \mathbf{p} , \mathbf{R} , and z follow unit root processes. Both the Dickey-Fuller and the Kwiatkowski et al. tests suggest that community income, six of the seven composite prices, and nine relative prices follow unit root rather than trend stationary processes. Both tests also suggest the relative price of beef is trend-stationary. The tests conflict for the dairy price index and the remaining nine relative prices. A test of the Joint Confirmation Hypothesis (JCH) of a unit root (Silvestre, Rossello, and Ortuno) confirmed the presence of a unit root in the dairy price index and in six of those remaining nine relative prices. A unit root could not be confirmed for the relative prices of butter, fresh vegetables, and eggs, and so these relative prices are considered trend stationary. The results in table 1 suggest unit root processes generate all composite prices, community income, and fifteen of the nineteen relative prices.

Table 2 reports the Engle-Granger test statistics (T_k) for each of the fifteen integrated relative price regressions. Each is specified as a function of an intercept, a time trend, community income, the five food-at-home, the away-from home, and the nonfood price indices. With the exception of processed fruits

Table 1. Unit Root Tests of Income and Group and Relative Prices

Null Hypotheses	$I(1) \tau_\tau$	$I(0) \eta_\tau$	$I(1) \text{ or } I(0)?$
R (Cereal and bakery)	-1.611 (8)	0.237 (6)*	I(1)
ρ (cereal)	-1.428 (8)	0.261 (6)*	I(1)
ρ (bakery)	-1.487 (8)	0.260 (6)*	I(1)
R (Meat)	-0.967 (5)	0.205 (6)*	I(1)
ρ (beef)	-3.246 (8)*	0.116 (6)	I(0)
ρ (pork)	-3.014 (6)	0.102 (5)	I(1) (JCH)
ρ (other meat)	-2.206 (5)	0.185 (6)*	I(1)
ρ (poultry)	-2.519 (6)	0.089 (5)	I(1) (JCH)
ρ (fish and seafood)	-2.152 (6)	0.136 (6)*	I(1)
R (Dairy)	-2.217 (6)	0.077 (6)	I(1)
ρ (fluid)	-2.575 (3)	0.112 (6)	I(1) (JCH)
ρ (butter)	-4.639 (8)*	0.201 (6)*	I(0) (JCH)
ρ (cheese)	-1.186 (2)	0.208 (6)*	I(1)
ρ (ice cream)	-2.678 (3)	0.111 (6)	I(1) (JCH)
R (Fruits and vegetables)	-1.751 (5)	0.152 (6)*	I(1)
ρ (fresh fruit)	-1.914 (6)	0.190 (4)*	I(1)
ρ (fresh vegetables)	-3.159 (8)	0.068 (1)	I(0) (JCH)
ρ (proc. fruit & veg)	-3.104 (8)	0.118 (1)*	I(1)
R (Other food at home)	-1.636 (7)	0.124 (6)*	I(1)
ρ (sugar and sweets)	-2.266 (6)	0.118 (6)	I(1) (JCH)
ρ (fats and oils)	-3.389 (5)*	0.146 (6)*	I(1) (JCH)
ρ (nonalcoholic bev)	-2.153 (2)	0.164 (6)*	I(1)
ρ (eggs)	-2.562 (6)	0.054 (5)	I(0) (JCH)
ρ (miscellaneous foods)	-2.160 (8)	0.180 (6)*	I(1)
R (Food away from home)	-1.847 (5)	0.290 (6)*	I(1)
R (Nonfood)	-0.928 (3)	0.239 (6)*	I(1)
z (Income)	-1.551 (8)	0.194 (4)*	I(1)
10% critical values:	$\tau_\tau^* = -3.167$	$\eta_\tau^* = 0.119$	$(\tau_\tau, \eta_\tau)^* = (-3.601, 0.073)$

Notes: Asterisk (*) denotes rejection of the null at the 0.10 level of significance. The test statistics of the null hypothesis of $I(1)$ (τ_τ) are the augmented Dickey and Fuller (1979) (*ADF*) *t*-values of the coefficient on the lagged level variable in the regression of the first-differences on a constant, a time trend, the lagged level, and lagged-differences of variables appended to the regression. The number of lags of first differences is reported in parentheses and determined by *SHAZAAM* 7.0. The second column (η_τ) reports test statistics developed by Kwiatkowski et al. (*KPSS*). They are sums of squared partial sums of residuals divided by an error variance estimator. The residuals are computed from a model in which the series is regressed on a constant and a time trend, and the error variance estimator is a Bartlett kernel weighted-sum of auto-covariances, with the automatic (Newey-West) bandwidth parameter reported in parentheses. The third column reports inference based on the Joint Confirmation of a Unit Root, and is used when the tests in the first and second columns conflict (Silvestre, Rossello, and Ortuno). The joint critical values $(-3.601, 0.073)$ represent the midpoint of critical values for fifty and hundred observations for the *ADF* and the *KPSS* (with Bartlett kernel) tests with trend. They are interpreted as follows. If the value of the *ADF* statistic (column 2) is less (greater) than -3.601 and the value of the *KPSS* statistic (column 3) is less (greater) than 0.073 then the series is considered (at the 0.90 level) stationary (integrated). Otherwise the series cannot be confirmed to be a unit root and is therefore considered to be stationary.

and vegetables, the each individual test failed to reject the null of spurious regression. Following Davis, Lin, and Shumway the family-wise test statistic of no cointegration is $\max |T_k| = 5.989$. For a 10% family-wise significance level, the (0.10/15) critical point of the distribution of this statistic under the null of no cointegration for each of the tests and for seventy-six observations is $T^* = 6.952$ (MacKinnon). Since $\max |T_k| < T^*$, the tests fail to reject the aggregation scheme. This suggests composite demand elasticities for this scheme accurately reflect the elasticities for the products that consumers actually purchase (Lewbel).

The system estimates are computed in three steps. First, compute estimates of Σ_{12}^0 and Σ_{22}^0

by applying the NSUR estimator to the system using the $M - 1$ identity matrix as the weight matrix in the first stage. Specifically, denote the first-stage residuals as $\mathbf{r}_t = [r_{1t}, \dots, r_{M-1,t}]$, and the vector of first differences of nonredundant regressors (with drift removed) as \mathbf{v}_t . From these compute the contemporaneous covariance matrix, $\mathbf{S}_{22}^0 = (1/T) \Sigma \mathbf{v}_t^* \mathbf{v}_t'^*$, where $\mathbf{v}_t^* = \mathbf{v}_t - \mathbf{c} \mathbf{v}_{t-1}$, where $\mathbf{c} = (\Sigma \mathbf{v}_{t-1} \mathbf{v}_{t-1}')^{-1} (\Sigma \mathbf{v}_{t-1} \mathbf{v}_t')$, and the cross-covariance matrix, $\mathbf{S}_{12}^0 = (1/T) \Sigma (\mathbf{r}_t \mathbf{v}_t'^*)$. Second, construct the transformed model as $\mathbf{W}_{jt}^* = \mathbf{W}_{jt} - \mathbf{s}_{120}^{(j)} (\mathbf{S}_{22}^0)^{-1} \mathbf{v}_t'$, where $\mathbf{s}_{120}^{(j)}$ denotes the j th row of \mathbf{S}_0^{12} . Third, apply NSUR to the transformed system of cointegrated regressions.

Table 2. Individual and Joint Tests of Spurious Regressions

Relative Price Regression	T_k
1. Cereal	-4.491(8)
2. Bakery	-4.415(8)
3. Beef	NC
4. Pork	-3.544(8)
5. Other meat	-3.318(5)
6. Poultry	-3.561 (7)
7. Fish and seafood	-2.436 (8)
8. Fluid	-3.471 (3)
9. Butter	NC
10. Cheese	-2.390 (7)
11. Ice cream	-4.658 (2)
12. Fresh fruit	-4.935(4)
13. Fresh vegetables	NC
14. Processed fruits and vegetables	-5.989 (8)
15. Sugar and sweets	-4.600 (8)
16. Fats and oils	-4.771 (5)
17. Nonalcoholic beverages	-3.273 (6)
18. Eggs	NC
19. Miscellaneous foods	-3.473 (8)
10% critical values:	
$T^* = -5.7381$ (individual tests)	
$T^* = 6.9521$ (family-wise test)	

Notes: The entries (T_k) are Engle-Granger tests of the null that the k th relative price and the vector of composite group prices and income are not cointegrated. The entries are augmented Dickey-Fuller tests of $I(1)$ residuals formed from regressing the k th relative price on each of the seven integrated group price indices (see table 1), income, a constant, and a time trend. The number of lagged first difference residuals included (in the residual regression) is reported in parentheses, and is determined by SHAZAM 7.0. The 0.10 critical values reported for the individual tests are based on seventy-six observations and eight integrated explanatory variables, so that $k = 9$ in MacKinnon. The 0.10 family-wise critical value of 6.952 is based on seventy-six observations, $k = 9$, and the (0.10/15) critical point.

Table 3 presents estimates of composite consumer demand elasticities. They are based on a rank three Θ matrix. The results yield relatively large estimates of income elasticities, although such results may be attributed to the very broad definitions of the composites. The results suggest the fruits and vegetable composite is the most price elastic and meat is the least price elastic. Except for the

meat composite, the results suggest the food-away-from-home composite is a gross substitute for the at-home food groups, and nonfood is a gross complement for all at-home food groups.

At this point we note that the above estimates are based on consumer-reported expenditure data rather than USDA's computed farm-based commercial disappearance data (e.g., Eales and Unevehr). The problem with using the commercial disappearance data in consumer demand analysis is these data provide information only on the physical amount of farm components in food. By ignoring the value that consumers place on the *mix* of food products, commercial disappearance ignores the fact that the mix of food products purchased has changed over time. Nelson shows that the CCT permits composite demand to be decomposed into a physical component and a quality component, where quality is a value measure of the mix of products purchased and where variations in quality reflect changes in the mix of products purchased over time. Reed, Levedahl, and Clark show this same decomposition follows from the GCCT, and provide evidence that consumers respond to changes in prices and income mostly by adjusting the mix of products purchased. Hence using commercial disappearance data as a proxy for food demand omits this important aspect of consumer demand for food.

Finally, there is interest in checking for weak separability. Under weak separability, the model errors of a composite demand system are not correlated with relative prices. Because the model errors are presumed to be stationary, a test for weak separability reduces to a test that the model errors are uncorrelated with stationary elements of \mathbf{p} . Table 1 suggests the relative prices for beef (p_b), butter (p_{bu}), fresh vegetables (p_v), and eggs (p_e) are stationary. If u_k denotes the residual of k th composite demand equation, we estimate

Table 3. Composite Demand Elasticities

	Cereal & Bakery (R_1)	Meats (R_2)	Dairy (R_3)	Fruits & Veggies (R_4)	Other Food (R_5)	Away (R_6)	Nonfood	Income (z)
Cereal and bakery	-0.606	0.036	-0.396	0.399	-0.673	0.182	-0.293	1.351
Meat	0.014	-0.605	0.257	-0.072	0.180	-0.736	-0.849	1.810
Dairy	-0.547	0.589	-0.861	-0.143	-1.260	1.321	-1.346	2.246
Fruits and vegetables	0.357	-0.108	-0.089	-0.979	-0.237	0.497	-1.042	1.601
Other food (home)	-0.337	0.176	-0.461	-0.125	-0.741	0.656	-0.207	1.038
Food away	0.049	-0.337	-0.276	0.154	0.344	-0.692	1.173	1.379
Nonfood	0.001	-0.002	-0.002	-0.008	-0.003	-0.045	-0.864	0.924

Table 4. Weak Separability Tests

Groups	W	P ($W > \chi^2$)
1. Cereal and bakery	59.37	<0.0001
2. Meat	51.73	<0.0001
3. Dairy	36.65	<0.0001
4. Fruit and vegetables	26.62	<0.0001
5. Other food at home	66.59	<0.0001
6. Food away	13.52	0.036

Notes: The entries (W) are Wald statistics associated with the null that relative prices are not related to the composite model errors. Each statistic is based on equation (13) in the text, and therefore each is distributed chi-square with six degrees of freedom. The third column reports probabilities of observing the reported level of W under the null of weak separability.

$$(13) \quad u_{kt} = \pi_{ko} + \pi_{k1}\rho_{bt} + \pi_{k2}\rho_{but} \\ + \pi_{k3}\rho_{vt} + \pi_{k4}\rho_{et} + \pi_{k5}(\rho_{bt}\rho_{but}) \\ + \pi_{k6}(\rho_{vt}\rho_{et}) + \xi_{ktj}.$$

for $k = 1, \dots, M - 1$ and test the null $\pi_{k1} = \pi_{k2} = \pi_{k3} = \pi_{k4} = \pi_{k5} = \pi_{k6} = 0$. The results presented in table 4 suggest this aggregation scheme cannot be based on weakly separable preferences.

Conclusions

One part of Lewbel's message is that the Generalized Composite Commodity Theorem may support a number of different aggregation schemes. Another part suggests this theorem could lead to improved estimates of consumer demand elasticities. This article represents an attempt to address both of these points.

Our results agree with previous studies that suggest data used in food demand analysis are generated from unit root processes so that tests for valid aggregation may reduce to tests for spurious regressions. We build on these studies by applying multi-comparison procedures to multiple rather than simple regression models. This simplifies testing and leads to a straightforward test of the aggregation scheme. Moreover, we choose a popular form to describe this composite food demand system, and show it can be treated as an estimable nonlinear system of cointegrated regressions. The demand elasticities for six broadly defined food categories appear to be reasonable, and tests reject weak separability.

Other results not reported here suggest the elementary food products chosen for this study could have been aggregated differently. While this may be symptomatic of the low power of residual tests for spurious regression, they

may also reflect the notion that the stochastic nature of the GCCT may support numerous aggregation schemes. This would suggest, for example, that demand and market analysis applied to nutrition-based aggregates such as USDA's food pyramid applies equally well to analysis based on the more traditional farm-based aggregates.

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